On Size and Magnetics: Why Small Efficient Power Inductors Are Rare

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***Abstract*—Of the three main component types needed in power converters—switches, capacitors and inductors—the most difficult to integrate on a semiconductor chip or in a planar package is the inductors. This difficulty arises partly from process compatibility challenges with magnetic materials, and is exacerbated by the fact that, because most types of electronics don't need inductors, there has been relatively little development effort. But a more fundamental challenge is the way magnetics performance scales with size. Capacitors and semiconductor devices can be made from thousands of small cells connected in parallel, but that approach would severely undercut the performance of magnetic components.**

Scaling relationships for magnetics are explored to demonstrate the inherent difficulty of small size and low profile magnetics. Cases considered include those with winding designs limited by skin and proximity effect and those constrained by efficiency and thermal dissipation. Small-scale magnetic components are typically limited by efficiency rather than heat dissipation. With efficiency constrained, and considering high frequency winding loss effects, it is shown that power density typically scales as the linear dimension scaling factor to the fifth power.

I. Introduction

Over the past half century, there has been astounding progress in miniaturization of electronics, primarily through increasing levels of integration on Si chips. For information handling, this miniaturization has come partly through reducing the amount of energy involved per bit. Miniaturization of power electronics is inherently more difficult, and the slower progress in power electronics has made power conversion an increasingly important bottleneck. A key part of the challenge for miniaturization and integration of power electronics is the need for magnetic components—inductors and transformers—which are neither required nor available in conventional information processing integrated circuits.

There are many reasons that it is difficult to integrate power magnetics, including the fact that the preferred materials and processes are different or even incompatible [1], [2]. Without discounting the importance of those issues, we explore here an even more fundamental issue that makes small, efficient power inductors challenging, regardless of the technology used: the way magnetics performance scales with physical size. We show that, in contrast to capacitors and semiconductors which scale down well, magnetic components' performance is best at larger sizes. This drives the need for custom magnetics design for particular applications, fully utilizing the available space: modular approaches combining smaller components or cells, as can be used with semiconductors and capacitors, could severely degrade magnetics performance. Nevertheless, it is possible to obtain good performance even in small packages, if the disadvantages of small sizes are understood and mitigation strategies are applied.

A key strategy towards miniaturization of power passives is the use of high frequencies, and the scaling with respect to frequency has been examined in, for example, [2]–[6]. Our goal in this paper is to narrow the focus to the scaling of performance with physical size, once a frequency has been selected. When miniaturization is desired, high frequencies will typically be used. Thus, high-frequency winding loss effects must be considered. Nonetheless, we start with a simple analysis neglecting high-frequency winding loss effects in Section II followed by an analysis of situations subject to these effects in Section III. We limit the discussion to magnetic-core components, and refer the reader to [3] for an analysis of scaling of air-core components. The analysis is similar that that in [6], but a different set of scenarios is examined.

Scaling laws are widely used in fluid dynamics, where dimensionless parameters such as Reynolds number are used to systematically account for the scaling and enable scale-model experiments that accurately reflect behavior of a larger system through "similitude." In biology, the concept is termed "allometry," and is used to explain otherwise surprising phenomena, such as the ability of ants to carry many times their body weight. Our title derives from two well known [7] and excellent [8] popular books on allometry in biology.

II. Low-frequency magnetic scaling

A. Constant flux and current density

A well known basic reference case for magnetics scaling neglects high-frequency winding loss effects. It is assumed that winding loss limits the spatial average of rms current density in the winding window to J_0, averaging over the copper area and the space between turns, such that J_0 reflects the impact of packing factor for the wire. It is also assumed that the flux density is limited to a value B_0, based either on losses or on saturation. To calculate the scaling of the VA handled by a transfomer or inductor subject to these limits, we note that voltage in a winding is proportional to the product of the number of turns in that winding N, the frequency f, and the flux linked by the winding, Φ. The flux is given by $\Phi = B_0 A_c$ where A_c is the cross-sectional area of the core. Similarly the current is given by $I = J_0 A_w / N$, where A_w is the area of the winding in question. Thus, VA is given by

$$VA = V \cdot I = (NfB_0 A_c) \left(\frac{J_0 A_w}{N} \right) = f(B_0 J_0)(A_c A_w) \quad (1)$$

The number of turns drops out, and the VA or power handling is proportional to the area product $A_c A_w$. If we scale all linear dimensions by a factor ϵ, the areas scale as ϵ^2 and the area product as ϵ^4, while the volume scales as ϵ^3. Thus, the power density scales as $\epsilon^4/\epsilon^3 = \epsilon$. Although there are many limitations to this applicability of this analysis, it shows that there is a fundamental advantage to larger-scale magnetics.

Note that (1) appears to indicate that power handling is proportional to frequency. However, that is assuming that the allowable flux density, B_0, is independent of frequency, which would only be true if it were limited only by saturation. In most practical cases, it is also limited by core loss, which depends on frequency. The product $fB_0(f)$ where $B_0(f)$ is the loss-limited allowable flux density as a function of frequency, is often described as the performance factor of a magnetic material, and can be useful in choosing a frequency of operation [5]. For our purposes, we consider frequency and performance factor to be fixed constants.

For the simple case described by (1), we can also easily see that a larger size component not only has higher power density, but also has higher efficiency, which we assess through the loss fraction, P_{loss}/VA. Given the assumption of constant loss density, the loss is proportional to ϵ^3, so combining that with (1) we have

$$\frac{P_{loss}}{VA} \propto \frac{\epsilon^3}{\epsilon^4} = \epsilon^{-1}. \quad (2)$$

The loss fraction is simply inversely proportional to the linear scaling factor.

From this analysis we can, for example, see a fundamental disadvantage of making a 100 W transformer by assembling 100 smaller transformers, handling 1 W each. With the same current and flux densities, each small transformer would need an area product 100 times smaller than the original, so the linear dimensions could be scaled down by a factor of $100^{1/4}$, or 3.16. This means the volume of each small transformer would be $100^{3/4}$ times smaller. One hundred of them would occupy $100 \cdot 100^{-3/4} = 100^{1/4}$, or 3.16, times more volume than the single-transformer solution, even in the ideal case. In practice, practical issues such as space devoted to terminations and insulation would make the 100-transformer array even worse. This is in sharp contrast to semiconductors and capacitors where arrays of small units work well with no theoretical degradation in power density. Note that the 100-transformer array, in addition to occupying three times the volume, also has three times higher loss.

B. Temperature rise limited scaling

Although the simple analysis of the base case is useful in explaining the fundamental advantage of larger scale magnetics, the allowable loss density in a very small component is much higher than the allowable loss density in very large component. Thus, we also consider a more realistic constraint of a maximum heat flow per unit surface area.

We now need to consider the scaling of loss with current density and flux density. The winding loss density is simply $J^2\overline{\rho}$ where $\overline{\rho}$ is the average resistivity accounting for packing factor and J is the spatial average of rms current density as before.

For core loss, the Steinmetz equation can be used. The commonly used extension of Steinmetz's original equation includes a frequency term, approximating losses as being proportional to f^α, but because we are not considering variation with frequency, we revert to the original form of the equation [9], $P_v = k\hat{B}^\beta$, where P_v is the loss per unit volume, $\hat{B}$ is the peak amplitude of the flux density, and k and β are empirical parameters that depend on frequency.

With loss per unit area fixed, the analysis in the Appendix shows that the VA capability scales as

$$VA \propto \epsilon^{3.5-1/\beta}. \quad (3)$$

Typical values of β are in the range of 2 to 3, resulting in an exponent in (3) between 3 and 3.17, with a typical value of 3.1 for a typical value of $\beta = 2.5$. Thus, we see that the power handling per unit volume is almost constant as the size is changed, still decreasing for small sizes, but not as much as with the constant loss density assumption. However, the efficiency now decreases faster than before as size gets small: The loss fraction scales as

$$\frac{P_{loss}}{VA} = \frac{\epsilon^2}{\epsilon^{3.5-1/\beta}} = \epsilon^{-1.5+1/\beta}. \quad (4)$$

The exponent ranges from -1 to -1.17 for β between 2 and 3.

C. Efficiency limited scaling

In both the base case and the temperature rise limited scaling, efficiency decreases at small sizes. Thus, the low efficiency, rather than the temperature rise, becomes the limiting factor for small components. With constant efficiency (i.e., constant loss fraction), the analysis in the Appendix shows that VA capability scales as

$$VA \propto \epsilon^{\frac{\beta}{\beta-2}+2+\frac{2}{\beta-2}+2} \propto \epsilon^{3+\frac{2\beta}{\beta-2}}. \quad (5)$$

For a typical value of $\beta = 2.5$, the exponent is 13, indicating that, with efficiency fixed, the VA capability changes very rapidly with linear dimensions. For a very small size, the VA needs to be extremely small to keep the efficiency constant. We can also state this in terms of volume—the VA varies with the 13/3 = 4.33 power of volume, such that a factor-of-two change in volume provides a factor-of-20 change in power capability. At higher values of β, the exponent in (5) decreases a bit, to 9 at $\beta = 3$. At low values of β, the exponent increases rapidly, and approaches ∞ as β approaches 2. That means that the VA varies very rapidly with size, or equivalently, that the efficiency is determined only by size, independent of the VA, in the limit of $\beta = 2$.

The fact that efficiency is independent of VA in the limit of $\beta = 2$ can be understood from the fact that in a linear system, losses are proportional to the square of the drive level, as is the case with winding loss. In the case that core loss also behaves linearly, the full system is linear and can operate with constant efficiency as VA is scaled up and down, assuming voltage and current are scaled together, or that the number of turns is adjusted accordingly.

III. Scaling with High-Frequency Winding Loss

To analyze the scaling of high-frequency magnetic components we must consider the impact of high-frequency winding loss, including skin effect and proximity effect losses. In windings with significant proximity-effect losses, or potential for significant proximity-effect losses, good design practice requires optimization [10], [11], so we assume that the winding design has been optimized in both the original design and the scaled design. However, the appropriate optimization may involve different constraints for different situations [5], [12]–[14], so we consider several scenarios. The simplest scenario to consider is one in which a strategy such as litz wire successfully reduces the ac resistance to be approximately equal to the dc resistance; in that case the scaling is identical to that in Section II, and the analysis there applies.

Also straightforward to consider is a scenario with a single-layer winding, thicker than a skin depth, such that current flows in a layer on the surface of the winding, one skin depth deep. The area of current flow is $A'_w = b\delta$ where b is the breadth of the winding and δ is the skin depth, and is proportional to ϵ rather than ϵ^2, because only b scales with ϵ. Thus, the scaling of NI with respect to the current density in that region, J, and with respect to ϵ becomes

$$NI \propto J\epsilon, \tag{6}$$

and the winding resistance is independent of ϵ. Winding loss scaling can be written as $P_w \propto N^2 I^2$ or

$$P_w \propto \epsilon^2 J^2. \tag{7}$$

In the case of a multilayer winding, using layers that are thin compared to a skin depth to reduce proximity effect losses, constraining the number of layers to a value p and using the optimum layer thickness for minimum ac resistance results in loss that is reduced by a factor $1/\sqrt{p}$ compared to the loss with a single-layer winding [13]. Thus, the scaling behavior in (6) and (7) is identical, even though the loss is smaller.

Another case to consider is using multiple thin layers or fine strands, with a constraint on the minimum strand diameter or layer thickness, and with the number of layers or strands chosen for minimum ac resistance. As shown in [13], this results in an improvement in ac resistance relative to a single-layer design by a factor $2t/(3\delta)$ where t is the layer thickness or, for litz strands, the effective layer thickness $t = 0.584d$, where d is the strand diameter. If t is considered to be a technological constraint, independent of the dimensional scaling factor ϵ, the result is the same scaling as in (6) and (7).

The final scenario that one might want to consider is a fixed number of strands of wire, including the case of using simple magnet wire, i.e., the number of strands set to one. This is unlikely to be the constraint of interest in practical high-frequency designs, so we do not fully analyze it, but only note that the result is ac resistance proportional to $\epsilon^{1/3}$, rather than independent of ϵ as in the other cases considered here.

We proceed to analyze the scaling of VA capability with respect to size, based on (6) and (7), with the understanding that the results apply to a single-layer winding, thicker than a skin depth; to an optimized multilayer winding with a constrained number of layers; or to an optimized multilayer winding with a constrained layer thickness or strand diameter. Because high-frequency effects make the current distribution in the winding non-uniform, the case of constant loss density is no longer possible, and so we analyze only the constraints of constant surface heat flux and constant efficiency.

A. Temperature rise limited scaling

The scaling of high-frequency magnetic components is considered with the maximum heat flow per unit surface area constrained. The derivation is similar to that in Section II-B, but, based on (7) the winding loss now scales with ϵ^2.

Equating the winding loss to the allowed winding loss $P_{allowed,w} = k_{\ell,w}\epsilon^2$ results in

$$\epsilon^2 J^2 \overline{\rho} = k_{\ell,w}\epsilon^2. \tag{8}$$

Thus J is independent of the scaling factor ϵ, and $NI \propto \epsilon$.

The core loss analysis is the same as in Appendix A, and (14) still applies: $N\Phi \propto \epsilon^{2-1/\beta}$. We find that the VA capability scales as

$$VA \propto \epsilon^{3-1/\beta}. \tag{9}$$

For typical values of β between 2 and 3 the exponent in (9) is between 2.5 and 2.67.

The resulting loss fraction is

$$\frac{P_{loss}}{VA} = \frac{\epsilon^2}{\epsilon^{3-1/\beta}} = \epsilon^{-1+1/\beta} \tag{10}$$

Based on typical values of β the exponent in (10) is between -0.5 and -0.67.

Assuming a maximum heat flow constraint, the power handling per unit volume of a high-frequency magnetic component actually improves as it is reduced in size. Despite the increase power handling density, the loss fraction associated with a maximum heat flux constraint indicates a serious degradation in efficiency, resulting in the need to examine efficiency limited scaling for the high-frequency case.

B. Efficiency limited scaling

Modifying the analysis in the Appendix based on the high frequency winding area A'_w scaling with ϵ and the effective winding volume $\mathcal{V}'_w$ scaling with ϵ^2, the VA capability becomes

$$VA \propto \epsilon^{\frac{\beta-1}{\beta-2}+1+\frac{1}{\beta-2}+2} \propto \epsilon^{3+\frac{\beta}{\beta-2}}. \tag{11}$$

This gives an exponent ranging from 6 for $\beta = 3$ to ∞ for $\beta = 2$. The exponent is smaller compared to that for

TABLE I
SUMMARY OF SCALING FACTORS. GENERAL RESULTS ARE FOLLOWED BY RESULTS FOR A TYPICAL VALUE OF THE STEINMETZ EQUATION EXPONENT $\beta = 2.5$

Constraint	VA	VA/Volume	$\mathbf{P_{loss}/VA}$
Low Frequency			
Loss density	ϵ^4	ϵ	ϵ^{-1}
Heat flux	$\epsilon^{3.5-1/\beta}$	$\epsilon^{0.5-1/\beta}$	$\epsilon^{-1.5+1/\beta}$
	$\approx \epsilon^{3.1}$	$\approx \epsilon^{0.1}$	$\approx \epsilon^{-1.1}$
Efficiency	$\epsilon^{3+\frac{2\beta}{\beta-2}}$	$\epsilon^{\frac{2\beta}{\beta-2}}$	Constant
	$\approx \epsilon^{13}$	$\approx \epsilon^{10}$	
High Frequency			
Heat flux	$\epsilon^{3-1/\beta}$	$\epsilon^{-1/\beta}$	$\epsilon^{-1+1/\beta}$
	$\approx \epsilon^{2.6}$	$\approx \epsilon^{-0.4}$	$\approx \epsilon^{-0.6}$
Efficiency	$\epsilon^{3+\frac{\beta}{\beta-2}}$	$\epsilon^{\frac{\beta}{\beta-2}}$	Constant
	$\approx \epsilon^{8}$	$\approx \epsilon^{5}$	
High Frequency Air-Core [3]			
Heat flux	ϵ^3	Constant	ϵ^{-1}

low-frequency scaling in (5) because, for large components, the available winding space is utilized less efficiently; A'_w becomes small compared to A_w. However, as was discussed in Section II-C, the VA capability still scales very rapidly as the linear dimensions are scaled for fixed efficiency, and as β approaches 2, the efficiency becomes independent of the VA.

IV. CONCLUSION

The results are summarized in Table I, including results from [3] for air-core inductors. Because an air-core inductor's loss fraction increases as size is reduced, it is not possible to scale them with constant efficiency. The table includes the general results as a function of the Steinmetz exponent, β, as well as example results for a typical $\beta = 2.5$. In most cases both efficiency and power density get worse at small scales.

As components get small, the efficiency constraint normally becomes dominant. The possible power density then drops precipitously with size, severely limiting the potential for miniaturization. The result is that it is advantageous to maximize the volume available for magnetic components, and to select circuits that use few magnetic components, each handling high VA, rather than many small components each handling small VA.

APPENDIX
DERIVATION OF SCALING FACTORS

A. For Section II-B

Equating winding loss to the allowed loss $P_{allowed,w} = k_{\ell,w}\epsilon^2$. results in $\mathcal{V}_w J^2 \overline{\rho} = k_{\ell,w}\epsilon^2$, where $\mathcal{V}_w$ is the volume of the winding, which leads to

$$J \propto \epsilon^{-0.5}. \tag{12}$$

Because $NI \propto J\epsilon^2$,

$$NI \propto \epsilon^{1.5}. \tag{13}$$

A similar analysis of core loss results in $\hat{B} \propto \epsilon^{-1/\beta}$ and thus

$$N\Phi \propto \epsilon^{2-1/\beta}. \tag{14}$$

Combining (13) and (14), we find the VA capability scales as

$$VA \propto \epsilon^{3.5-1/\beta}. \tag{15}$$

B. For Section II-C

Assuming winding losses proportional to VA results in

$$\mathcal{V}_w J^2 \overline{\rho} \propto f\hat{B}JA_wA_c \tag{16}$$

$$\frac{J}{\hat{B}} \propto \epsilon. \tag{17}$$

Similarly, assuming core loss proportional to VA results in

$$\mathcal{V}_c k\hat{B}^\beta \propto f\hat{B}JA_wA_c, \tag{18}$$

$$\frac{J}{\hat{B}^{\beta-1}} \propto \frac{1}{\epsilon}. \tag{19}$$

Solving (17) and (19) simultaneously, we get $\hat{B} \propto \epsilon^{2/(\beta-2)}$ and $J \propto \epsilon^{\beta/(\beta-2)}$. As before, $NI \propto J\epsilon^2$ and $N\Phi \propto \hat{B}\epsilon^2$, which leads to

$$VA \propto \epsilon^{\frac{\beta}{\beta-2}+2+\frac{2}{\beta-2}+2} \propto \epsilon^{3+\frac{2\beta}{\beta-2}}. \tag{20}$$

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