A UNIFYING DERIVATION OF SWITCHING DC-DC CONVERTER TOPOLOGIES*

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ABSTRACT

A canonical switching cell is proposed from which the three basic DC-DC converter topologies can be derived. In addition, it is possible to perform topological transformations which yield apparently new topologies. Upon examination, however, these topologies may prove to be identical to existing basic topologies. An optimum topology converter recently described by Ćuk, et al., is shown to be a transformation of a basic topology.

THE CANONICAL CELL

The canonical switching cell is defined in Figure 1 where the inductor current is assumed to be continuous. The capacitor, C, is included to provide an AC short circuit between terminals 1 and 2 so that the external circuit need not carry discontinuous currents.

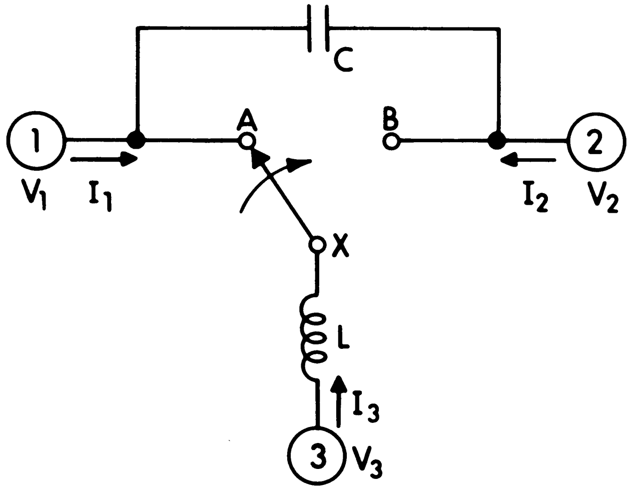

FIG. 1 CANONICAL SWITCHING CELL

In order to analyze this three-terminal network in the most general way, it will be assumed that none of the terminals is grounded. Voltages will be measured with respect to an external ground reference. The switch is assumed to be in position A for time DT and in position B for time D'T, where T is one period of operation and D + D' = 1. Conservation of charge and power yield:

$$I_1 + I_2 + I_3 = 0 \quad (1)$$

$$V_1 I_1 + V_2 I_2 + V_3 I_3 = 0 \quad (2)$$

Lossless components have been assumed and voltages and currents are steady-state DC values with no superimposed ripple. Averaging the currents yield:

$$I_1 = -DI_3 \quad (3)$$

$$I_2 = -D'I_3 \quad (4)$$

Substituting (3) and (4) into (2) yields:

$$-DV_1 - D'V_2 + V_3 = 0 \quad (5)$$

from which all voltage transfer functions can be derived. Substitution of (3) and (4) into (1) yield:

$$-D - D' + 1 = 0 \quad (6)$$

which checks the equations.

DOWN-CONVERTER

The classical down-converter is obtained by considering terminal 1 to be the input while terminal 3 is the output. Setting $V_2 = 0$ in (5) yields:

$$-DV_1 + V_3 = 0 \quad (7)$$

or

$$\frac{V}{V_g} = \frac{V_3}{V_1} = D \quad (8)$$

where V is the output voltage and V_g is the input voltage.

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In this case the switch is realized by a diode from X to B and a transistor from X to A, as shown in Figure 2. Note that the output current is continuous but contains the inductor ripple current. The input voltage will be essentially constant with only the capacitor ripple voltage present. The input current will contain a small continuous triangular ripple current if the capacitor impedance is low compared to the source impedance.

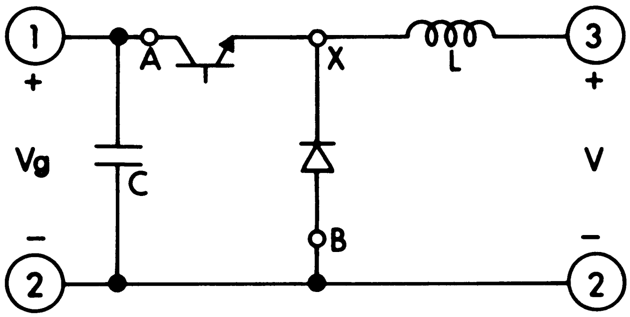

FIG. 2 DOWN-CONVERTER

UP-CONVERTER

An up-converter is realized when terminal 3 is the input and terminal 1 is the output. Again, terminal 2 is assumed to be grounded with $V_2 = 0$ and:

$$\frac{V}{V_g} = \frac{V_1}{V_3} = \frac{1}{D} \quad . \qquad (9)$$

The practical configuration of an up-converter is shown in Figure 3 where the switch from A to X has been replaced by a diode and the switch from B to X by a transistor. The output voltage is essentially constant with only the capacitor ripple voltage present while the output current contains a small continuous triangular ripple current if the capacitor impedance is low compared to the load impedance.

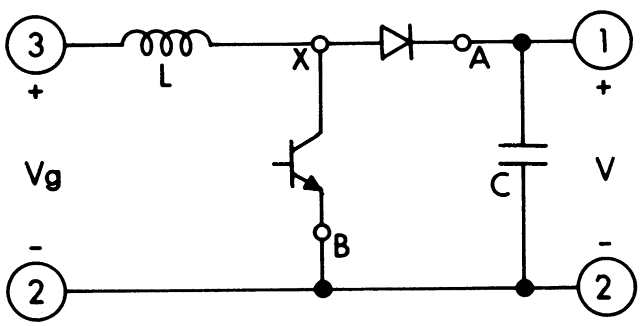

FIG. 3 UP-CONVERTER

BIDIRECTIONAL-CONVERTER

From the above descriptions, we note that a bidirectional-converter may be realized with the topology of Figures 2 and 3 through the use of bidirectional switches in place of the diode and transistor shown. Thus the up- and down-converters are both special cases of the generalized bidirectional-converter. For the general case, terminal 1 is always at a higher voltage than terminal 3 ($V_2 = 0$), and the direction of power flow is determined by the external circuit.

BUCK-BOOST CONVERTER

A buck-boost converter can be realized using terminal 1 as the input and terminal 2 as the output, as shown in Figure 4.

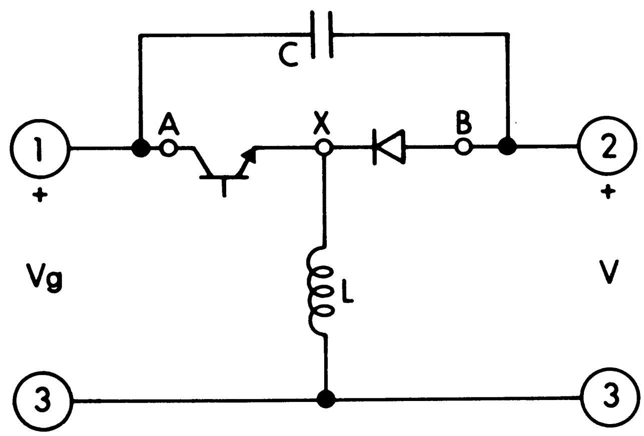

FIG. 4 BUCK-BOOST CONVERTER

For this case with $V_3 = 0$ in equation 5 we have:

$$DV_1 + D'V_2 = 0 \qquad (10)$$

and

$$\frac{V}{V_g} = \frac{V_2}{V_1} = -\frac{D}{D'}, = -\left(\frac{D}{1 - D}\right) \quad . \qquad (11)$$

Note that an appropriate substitution of devices for the switches has been made. This circuit can also be considered bidirectional, as described above, with the relationship between V_1 and V_2 as stated.

DYNAMIC TRANSFER FUNCTIONS

An analysis of the dynamics of a circuit configuration involving the canonical switching cell shown in Figure 1 is greatly simplified by the fact that only one dynamic transfer function need be developed. This is a consequence of the algebraic relationships between the terminal parameters (equations 1 and 2) and the assumption of bidirectional switches which can be replaced by transistors and diodes in the final configurations.

TERMINAL CURRENTS

None of the topologies thus far discussed has discontinuous terminal currents if the capacitor is included in the basic cell and capacitor impedance and external impedances are appropriately related. Note that the voltage across the inductor is in all cases a rectangular wave with duty cycle D (or D' = 1 - D) and an average DC value of zero. Thus the inductor current will contain a triangular ripple component.

The buck-boost converter is particularly interesting when one asks the question: "Where does the inductor ripple current circulate?" If the source AC impedance is low or zero, all the inductor ripple current comes from the source and the output ripple current can be zero except for the second-order effect of capacitor voltage ripple. If the load AC impedance is zero, then the input current ripple can be zero, excepting effects due to capacitor voltage ripple. One can now generalize that the inductor current ripple divides inversely proportional to the source and load AC impedances. Note that the only difference between the buck-boost converter presented here and those usually seen is in the placement of the capacitor between input and output.

One further question can be posed. In the case where the inductor ripple current circulates through the source because the load has no capacitor bypass, can the size of the capacitor (Figure 4) be adjusted so that the output current ripple due to capacitor voltage ripple cancels? The circuit operation reveals that these ripple currents and input current ripple can never cancel but only approach zero as L and C approach infinity.

TOPOLOGICAL TRANSFORMATIONS

A new coupled-inductor optimum topology converter, which has been described by Ćuk, et al., (1, 2), is shown in Figure 5. These studies conclude that a coupled inductor approach is desirable and that a turns ratio of 1:1 is required. The coupled inductors can be modeled by leakage inductances, a magnetizing inductance, and an ideal transformer, as shown in Figure 6. From this representation, the circuit of Figure 6 can be shown to be functionally identical to the classical buck-boost configuration of Figure 4. Two methods may be employed to demonstrate this identity.

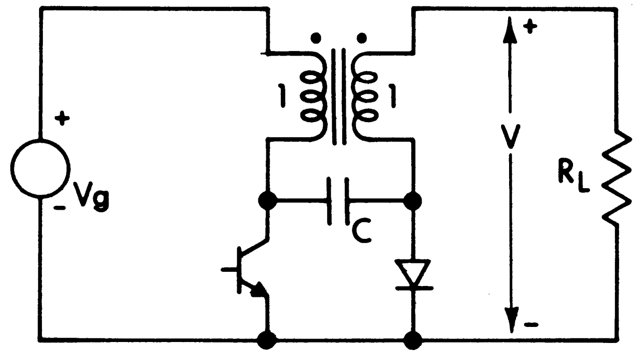

FIG. 5 OPTIMUM TOPOLOGY CONVERTER

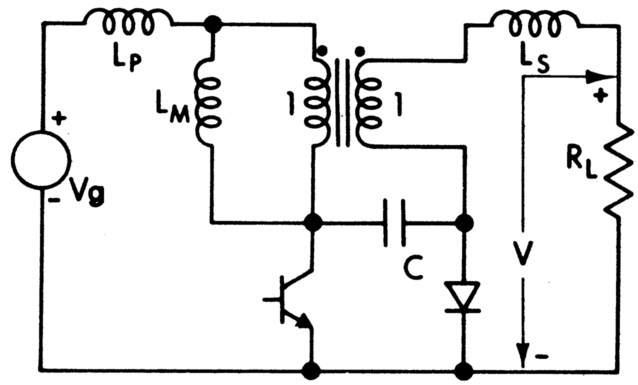

FIG. 6 MODEL

Method 1 - start by moving the right side ideal transformer winding clockwise past L_S and R_L to a position adjacent to the diode. Similarly, move the left ideal transformer winding and parallel inductance (L_m) counter-clockwise past L_P and V_g. The result is shown in Figure 7. The transformer, which is clearly redundant, can now be eliminated, as shown in Figure 8. The resultant circuit is identical to the original circuit of Figure 4 with the addition of the two leakage reactances to the source and load impedances.

A second method for demonstrating the equivalence is as follows:

Method 2 - in Figure 6 note that the voltage across the capacitor is not changed by sliding it upward through the ideal transformer to the point between the two leakage inductors. Since the capacitor voltage, and thus current, as well as the transistor current (equal to the generator current less the capacitor current) are unchanged, the movement of the capacitor has not altered the operation of the circuit. With this change plus the interchange of the positions of the ideal transformer/shunt inductor (L_m) combination with the transistor and diode switches, we arrive at the circuit shown in Figure 7.

From Figure 7 we then eliminate the unnecessary ideal transformer thus leaving the final circuit of Figure 8.

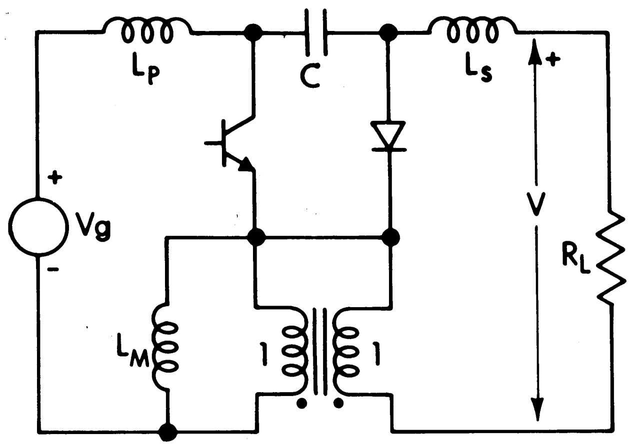

FIG. 7 TRANSFORMED CIRCUIT

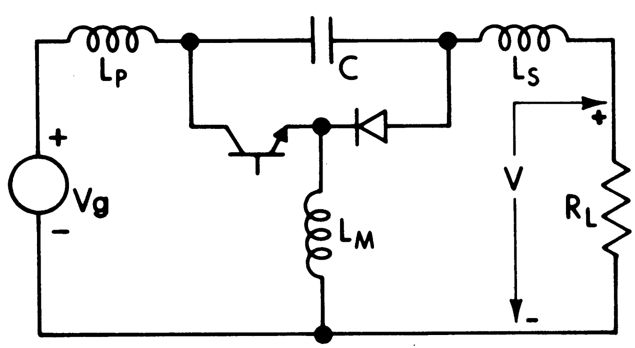

FIG. 8 FINAL TRANSFORMATION

At this point some comparisons can be made between the classical buck-boost configuration and the Ćuk coupled-inductor optimum topology. If the inductor/transformer remains physically the same size, then the resultant losses remain constant through the topological transformations. Thus neither circuit appears to have an advantage over the other. The statement that energy transfer is capacitive is also seen to be untenable.

CONDITIONS FOR ZERO CURRENT RIPPLE

The questions of the size and path of the current ripple can also be investigated with the aid of the circuit of Figure 8. If L_p is made zero and L_s finite, then the inductor ripple current circulates in the source and the output current ripple is zero (except that component due to the capacitor voltage ripple). If a filter capacitor is provided across R_L, then the reverse is true if L_s is zero and L_p finite. The necessary conditions will be derived using the same notation as used by Ćuk.

$$L_1 = L_p + L_m \tag{12}$$

$$L_2 = L_s + L_m \tag{13}$$

$$n = \sqrt{\frac{L_1}{L_2}} = \sqrt{\frac{1 + \frac{L_p}{L_m}}{1 + \frac{L_s}{L_m}}} \tag{14}$$

$$k = \frac{Lm}{\sqrt{L_1 L_2}} = \sqrt{\frac{1}{\left(1 + \frac{L_p}{L_m}\right)\left(1 + \frac{L_s}{L_m}\right)}} \tag{15}$$

where n and k are the apparent turns ratio and coupling constants, respectively.

For zero output ripple, $L_p = 0$ thus

$$n = k = \frac{1}{\sqrt{1 + \frac{L_s}{L_m}}} . \tag{16}$$

For zero input ripple, $L_s = 0$ thus

$$n = \frac{1}{k} = \sqrt{1 + \frac{L_p}{L_m}} . \tag{17}$$

Since the classic buck-boost converter has $L_p = L_s = 0$, it is equal to the best-case optimum converter. Note that output voltage ripple is directly related to the size of the capacitor. In other words, the capacitor cannot be made arbitrarily small without paying an output voltage ripple penalty.

OBSERVATIONS

The unique properties of the "continuous" buck-boost converter are a consequence of the location of the filter capacitor which provides a path so that input and output currents need not be discontinuous. From this observation we are lead to ask: Can we provide the same property for an "isolated" flyback type converter? In addressing this question we will take the turns ratio to be unity for convenience. Figure 9 shows a typical flyback circuit in which a current transformer has been added to couple the input and output filter capacitors so that the input current does not have a discontinuity when the transistor is turned off (or on). This circuit should have continuous input and output currents. Further, we are free to transform the configuration in a manner similar to that previously employed in reaching Figure 8. In particular, the capacitor connections can be slid through the transformer as discussed above to yield an isolated Ćuk-type converter.

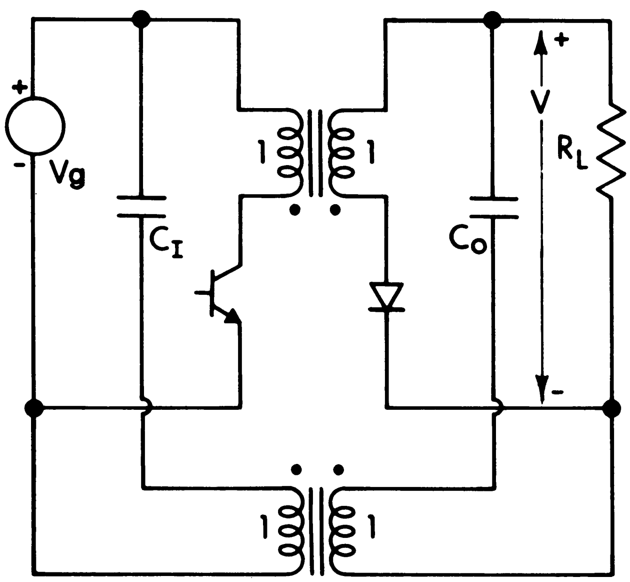

FIG. 9 CONTINUOUS-CURRENT FLYBACK CONVERTER

In the former case (Figure 9), the main transformer has higher I^2R losses but the current transformer supports almost no volt-seconds. In the latter case (isolated Ćuk circuit), the main transformer has less I^2R losses but the current transformer now supports volt-seconds equal to that of the main transformer, which probably increases the I^2R losses in the current transformer.

OTHER TOPOLOGICAL TRANSFORMATIONS

By splitting the inductor, transposing devices and windings, and sliding the capacitor through the transformer, the up- and down-converters can be similarly transformed to Ćuk configurations. Figures 10 and 11 show the final forms.

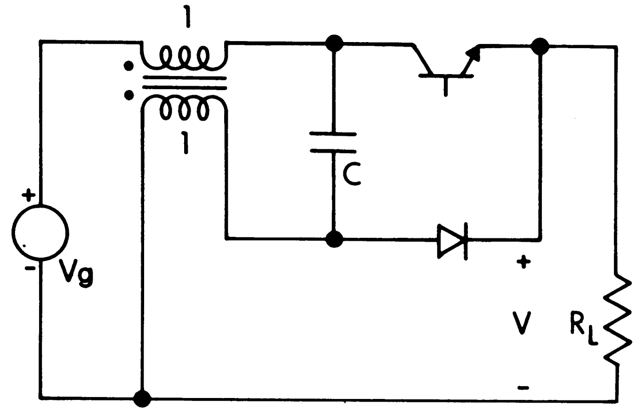

FIG. 10 DOWN-CONVERTER

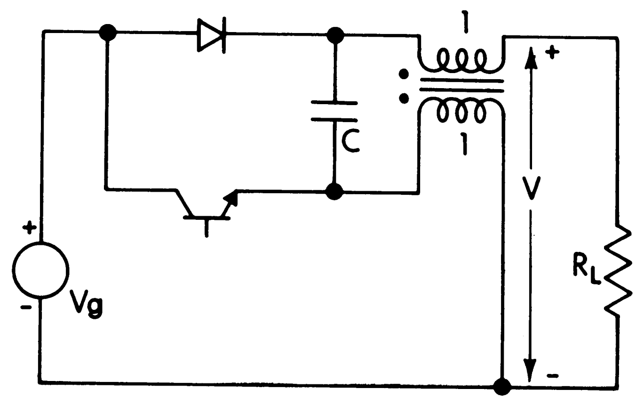

FIG. 11 UP-CONVERTER

SUMMARY

It has been shown that all three classical switching converter circuits can be derived from a single canonical switching cell. Further, recently published new topologies are shown to be electrically equivalent to existing well-known circuits. Component stresses and losses are therefore equal and the new topologies would exhibit no advantages.

REFERENCES

(1) S. Ćuk & R. D. Middlebrook, "Coupled-Inductor and Other Extentions of a New Optimum Topology Switching DC-to-DC Converter," IEEE Industry Applications Society Annual Meeting, Los Angeles, CA, 2-6 October 1977.

(2) S. Ćuk & R. W. Erickson, "A Conceptually New High-Frequency Switched-Mode Power Amplifier Technique Eliminates Current Ripple," Powercon 5, 1978, San Francisco, CA.